PG – 389

III Semester M.Sc. Examination, December 2016
(CBCS)
MATHEMATICS
M303 T : Fluid Mechanics

Time : 3 Hours | Max. Marks : 70

Instruction : *Answer **any five** full questions.*

1. a) Define an isotropic tensor. If a_{ij} are components of an isotropic tensor then show that $a_{ij} = \alpha\delta_{ij}$ for some scalar α.

   b) State and prove Stokes's theorem for a tensor field $\underset{\sim}{A}$. **(8+6)**

2. a) Explain briefly :

   i) Continuum hypothesis.

   ii) Lagrangian and Eulerian descriptions of motion.

   iii) Path lines, stream lines and vortex lines.

   b) Establish the Reynolds transport formula and hence deduce the expression for the rate of change of a material volume. **(9+5)**

3. Derive the field equations for conservation of linear momentum and energy. **14**

4. a) Establish Euler's equation of motion.

   b) Find the pressure distribution for a velocity field $\vec{q} = k(x^2 - y^2)\hat{i} - 2kxy\hat{j}$ (k = constant) which satisfies the Navier-Stokes equation for an incompressible fluid in the absence of body force. **(6+8)**

5. a) Derive the Helmholtz vorticity equation and stating the assumptions made. Deduce that $\vec{w}/\rho$ = constant for a travelling fluid element.

   b) Define impulsive motion. Derive the general equation of impulsive motion and stating the conditions. Show that the impulsive pressure is harmonic. **(7+7)**

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6. a) Define : complex potential, source, sink and doublet. For a two-dimensional flow field given by $\psi = xy$, show that the flow is irrotational. Also, find the velocity potential, streamlines and potential lines.

   b) State and prove the Mitne-Thomson circle theorem. **(8+6)**

7. Obtain the velocity distribution for

   i) Generalised plane Couette flow.

   ii) Hagen-Poiseuille flow. **(7+7)**

8. a) Discuss the velocity distribution for Stokes's second problem by deriving an expression for the velocity field.

   b) Stating the assumptions made, show that the rate of energy dissipation due to viscosity of the fluid is $W = \mu \int_V w^2 \, dV$, where the quantities have their usual meaning. **(8+6)**